2026 USAAAO Third Exam
(Data Analysis + Observation)

1 Instructions (Please Read Carefully)

The test must be completed within 2 hours (120 minutes) and the maximum number of points is **230 points**. The point breakdown per problem is as follows.

Problem	P1	P2	P3	P4	**Total**
Points	55	75	46	54	**230**

Please solve each problem on a blank piece of paper and mark the number of the problem at the top of the page. Some problems may require a designated answer sheet; these will be provided. The contestant's full name in capital letters should appear at the top of each solution page. If the contestant uses scratch papers, those should be labeled with the contestant's name as well and marked as "scratch paper" at the top of the page. Scratch paper will not be graded. Partial credit will be available given that correct and legible work was displayed in the solution.

This is a written exam. Contestants can only use a scientific calculator (non-programmable and non-graphing) for this exam. A table of physical constants will be provided. **Discussing the problems with other people is strictly prohibited in any way until the end of the examination period on June 25, 2026.** Receiving any external help during the exam is strictly prohibited. This means that the only allowed items are: a calculator, the provided table of constants, a pencil (or pen), an eraser, blank sheets of papers, and the exam. No books or notes are allowed during the exam. Students must be proctored during the entire duration of the exam.

This exam sheet and your answer sheets (including scratch papers) should be returned to your proctors once the exam ended. Your proctors should upload your answer sheets to the Google Form provided to them.

We acknowledge the following people for their contributions to this year's practical exam:

Ferdinand, Vincent Bian

After reading the instructions, please make sure to sign, affirming that:

1. All work on this exam has been completed by me.
2. I took this exam under the supervision of a proctor.
3. I did not receive any external help beyond the materials provided.
4. I will not discuss the contents of this exam with anyone until June 24, 2026.
5. Failure to follow these rules will result in disqualification from the exam.

Student Signature: ______________________ Date: ______________________________

1. **(55 points)** *Type II Supernova*

   In April 2024, the Asteroid Terrestrial-impact Last Alert System (ATLAS) reported the emergence of a rapid brightening object in the constellation Hydra. Follow-up observations confirmed that the transient was a nearby Type II supernova located at ~ 7.2 Mpc, one of the closest supernovae of the decade. Due to its proximity, the supernova, named SN 2024ggi, has been well-characterized in multiple wavelengths. It was revealed that the expanding supernova shock was moving outward at a nearly constant velocity of 10,000 km/s. Prior to the explosion, the progenitor star had been losing mass through a slow stellar wind with a velocity of 20 km/s, creating a dense circumstellar medium (CSM) around the star.

   When the rapidly expanding shock collides with this surrounding material, the gas is heated to temperatures of millions of kelvin and emits strongly in X-rays. As such, X-ray observations are the best tracer to constrain supernovae properties. X-ray observations in 2024 revealed the supernova's unabsorbed/intrinsic fluxes and emission measure[1] (EM) and can be summarized in the table below.

   Table 1: Observed unabsorbed flux and emission measure of the supernova

Time since explosion (days)	Unabsorbed Flux (erg s^{-1} cm^{-2})	EM (cm^{-3})
5.3	7.11×10^{-13}	2.75×10^{62}
10.9	4.98×10^{-13}	9.31×10^{61}
15.0	2.54×10^{-13}	6.08×10^{61}
16.2	2.85×10^{-13}	4.76×10^{61}
24.5	1.66×10^{-13}	3.76×10^{61}
34.8	1.31×10^{-13}	1.84×10^{62}
43.2	1.02×10^{-13}	1.07×10^{62}
55.0	0.75×10^{-13}	2.80×10^{61}
85.4	0.55×10^{-13}	2.16×10^{61}

   In this problem, we are trying to find out the brightness decay rate of a supernova and calculate the mass-loss rate of the supernova progenitor through the shock interactions with the circumstellar medium (CSM). For this problem, please report the final answer using time in days and other parameters in cgs (centimeter-gram-seconds) units.

   (a) **(20 points)** Plot the luminosity vs time (in the provided grid paper). The luminosity vs time has a power-law relation, i.e. $L = At^{\alpha}$, please estimate A and α to get the relation and draw the relation in the plot too. *Hint: it may be useful to draw a log-log plot.*

   (b) **(23 points)** As the supernova explosion expands, the surrounding CSM of the supernova is being affected. Brethauer (2022) derived that the unshocked CSM density is expressed by:

   $$\rho_{\text{CSM}} = \frac{m_p}{4}\sqrt{\frac{2 \times \text{EM} \times \mu_e \times \mu_I}{V_{\text{FS}}}}$$

   where m_p is the mass of proton, μ_e is the mean molecular weight per electron and can be assumed to be $\mu_e = 1.14$, μ_I is the mean molecular weight per ion and can be assumed to be $\mu_I = 1.24$, and $V_{\text{FS}} \approx \frac{4}{3}\pi\left((1.2 \cdot R_{\text{shock}})^3 - R_{\text{shock}}^3\right)$ is the volume of the forward shock. Now, find the unshocked CSM densities and plot them as a function of shock radius (in the provided grid paper). The density vs radius also has a power-law relation, i.e. $\rho = Br^{\beta}$, please estimate B and β to get the relation and draw the relation in the plot too. *Hint: it may be useful to draw a log-log plot.*

   (c) **(12 points)** Based on your previous results, please find the mass-loss rate and uncertainty of the supernova progenitor in $M_\odot$/yr. (*Hint: What is the relationship between density, radius, wind speed, and mass-loss rate? Disregard your fitting results in part b*)

2. **(75 points)** *Travelers in Outer Space*

[1] Emission measure (EM), usually used in X-ray and Extreme Ultraviolet astronomy, is the number density of electron times the number density of ion integrated over volume of the region. In other words: $\text{EM} = \int n_e n_I \, dV$.

Esker, Chert, and Gabbro are traveling in a spaceship in the Solar System when their spaceship explodes! Fortunately, everyone makes it to an escape pod in time and escapes safely, but they end up all scattered in different locations.

Note: For all questions asking you to compute a numerical answer from the data provided, include the uncertainty in your answer.

(a) (25 points total) Esker's escape pod ends up in orbit around some planet in the Solar System. They gaze longingly at Earth through a telescope, and every day they measure the apparent angular diameter of the Earth. Unfortunately, they don't have any precise instruments, so they can only measure the *relative* angular size of the Earth. You may assume that the absolute angular size of Earth is small.

The following is a table of the relative size over time. *The uncertainty in each measurement is 0.05.*

Table 2: Apparent relative angular diameter of Earth as seen from Esker's pod (arbitrary units)

Day	Rel. size	Day	Rel. size	Day	Rel. size
0	0.87	440	3.99	930	1.46
63	0.78	462	3.26	989	2.68
112	0.59	478	2.57	1004	3.34
169	0.77	543	1.28	1030	4.02
219	0.72	592	0.96	1045	3.54
284	0.87	653	0.96	1062	2.74
325	1.04	700	0.64	1079	2.12
392	2.35	756	0.62	1144	1.02
402	2.58	824	0.69	1190	0.87
418	3.33	869	0.98	1249	0.73

(i) **(5 points)** Let d_{near} be the closest distance between Earth and Esker, and let d_{far} be the farthest distance. What is the ratio $\frac{d_{\text{far}}}{d_{\text{near}}}$?

(ii) **(10 points)** What is the period of Esker's motion relative to the Earth?

(iii) **(10 points)** What is the radius of Esker's orbit around the sun? Where is Esker?

(b) (25 points total) Chert's escape pod lands on a rocky body in the Solar System, and they can't find Earth! Fortunately, they have a very accurate telescope, a star-chart computer, and a gyroscope that locks their coordinate frame to Earth's equatorial axes.

Chert measures the RA and declination of many bright stars and compares each measurement to the catalog value from Earth at the same date. The resulting angular offsets measured in *milli-arcseconds*, caused by the parallax of Chert's location relative to Earth, are listed below in Table 3 along with each star's distance. The uncertainty in each measurement is 0.001 mas (it is a *very* precise telescope).

Ignore aberration caused by orbital motion.

(i) **(5 points)** Suppose Chert is a distance r from Earth, and measuring the apparent location of a star which is a distance D from Earth (you may assume $D \gg r$). What are the possible values Chert might get for the angular distance θ between the apparent positions for that star on Earth vs at Chert's location?

(ii) **(12 points)** Plot θ vs $\frac{1}{D}$. Using this plot, determine how far Chert is from the Earth.

(iii) **(3 points)** Why do more of the data points lie closer to the maximum possible angular displacement than the minimum possible angular displacement?

(iv) **(5 points)** Where is Chert, and why might Earth not be visible from their location?

(c) **(25 points)** Gabbro's escape pod somehow ends up in orbit around another star! Just like Chert, they have a telescope, a star-chart computer, and a gyroscope that locks their coordinate frame to Earth's equatorial axes.

They measure the right ascension and declination of a few known stars and compare it to the equatorial coordinates of those stars when observed from Earth.

Where is Gabbro?

You are provided with an answer sheet with a gnomonic projection of the celestial sphere.

Hint: lines in a gnomonic projection correspond to great circles on the sphere

Table 3: Chert's stellar parallax measurements

Star	**Day**	θ **(mas)**	D **(ly)**	**Star**	**Day**	θ **(mas)**	D **(ly)**
Aldebaran	0	0.123	65	Dschubba	15	0.020	402
Tianguan	2	0.020	417	Tania	15	0.024	249
Furud	2	0.025	336	Chertan	16	0.020	178
Mirach	3	0.019	199	Shaula	17	0.009	703
Ankaa	4	0.096	77	Ascella	17	0.098	89
Fawaris	4	0.046	171	Seginus	18	0.081	85
Scheat	4	0.030	199	Hadar	22	0.013	525
Fulu	5	0.011	597	Porrima	22	0.205	39
Menkalinan	5	0.066	82	Izar	22	0.038	210
Alkaphrah	7	0.015	423	Unukalhai	22	0.074	73
Zosma	8	0.139	58	Matar	23	0.037	215
Meissa	9	0.005	1056	Sargas	23	0.010	272
Aludra	9	0.003	3198	Kitalpha	24	0.030	186
Gomeisa	11	0.022	170	Dalim	25	0.172	46
Cor	12	0.059	110	Azha	26	0.058	133

Table 4: Apparent positions of stars as seen from Earth and from Gabbro's pod

Star	**RA (Earth)**	**Dec (Earth)**	**RA (Gabbro)**	**Dec (Gabbro)**	**Dist (ly)**
Nekkar	$15^{h}01.95^{m}$	$+40^{\circ}\,23.44'$	$15^{h}21.41^{m}$	$+30^{\circ}\,09.67'$	219
Dubhe	$11^{h}03.73^{m}$	$+61^{\circ}\,45.07'$	$12^{h}52.67^{m}$	$+50^{\circ}\,51.11'$	124
Tania Australis	$10^{h}17.10^{m}$	$+42^{\circ}\,54.87'$	$11^{h}26.35^{m}$	$+32^{\circ}\,42.87'$	134
Ruchbah	$01^{h}25.81^{m}$	$+60^{\circ}\,14.12'$	$23^{h}00.46^{m}$	$+53^{\circ}\,12.78'$	99
Capella	$05^{h}16.69^{m}$	$+45^{\circ}\,59.94'$	$06^{h}29.42^{m}$	$+39^{\circ}\,47.67'$	43
Muscida	$08^{h}30.27^{m}$	$+60^{\circ}\,43.11'$	$09^{h}39.48^{m}$	$+60^{\circ}\,25.18'$	184
Polaris	$02^{h}31.78^{m}$	$+89^{\circ}\,15.85'$	$17^{h}49.74^{m}$	$+86^{\circ}\,22.46'$	431
Tania Borealis	$10^{h}22.33^{m}$	$+41^{\circ}\,29.97'$	$10^{h}58.42^{m}$	$+36^{\circ}\,30.60'$	249
Algol	$03^{h}08.17^{m}$	$+40^{\circ}\,57.34'$	$01^{h}52.16^{m}$	$+32^{\circ}\,28.99'$	93
Menkalinan	$05^{h}59.53^{m}$	$+44^{\circ}\,56.85'$	$06^{h}41.64^{m}$	$+42^{\circ}\,50.87'$	82
Mirach	$01^{h}09.73^{m}$	$+35^{\circ}\,37.25'$	$00^{h}30.19^{m}$	$+29^{\circ}\,41.37'$	199
Grumium	$17^{h}53.53^{m}$	$+56^{\circ}\,52.35'$	$17^{h}41.57^{m}$	$+34^{\circ}\,59.80'$	111
Schedar	$00^{h}40.51^{m}$	$+56^{\circ}\,32.24'$	$23^{h}47.19^{m}$	$+53^{\circ}\,28.47'$	229

3. **(46 points)** *Stars in the Telescope*

Please answer this question in the provided Question 3 Answer Sheet

Berwyn, an amateur astronomer, is observing at a random star in the night sky through his telescope. He first positioned the star in the center of his telescope's field of view (FoV) and labeled some of the stars as seen in Figure 5a. Since the telescope is not tracking the sky, the star slowly drifts across the telescope's FoV due to Earth's rotation. After 81.0 seconds, the telescope FoV looks like Figure 5b.

Berwyn then consults with his friends on his observations and gains additional information:

- Star 1 has magnitude 7.70.
- Star 2 has magnitude 9.8.
- Star 4 has magnitude 10.25 and coordinates

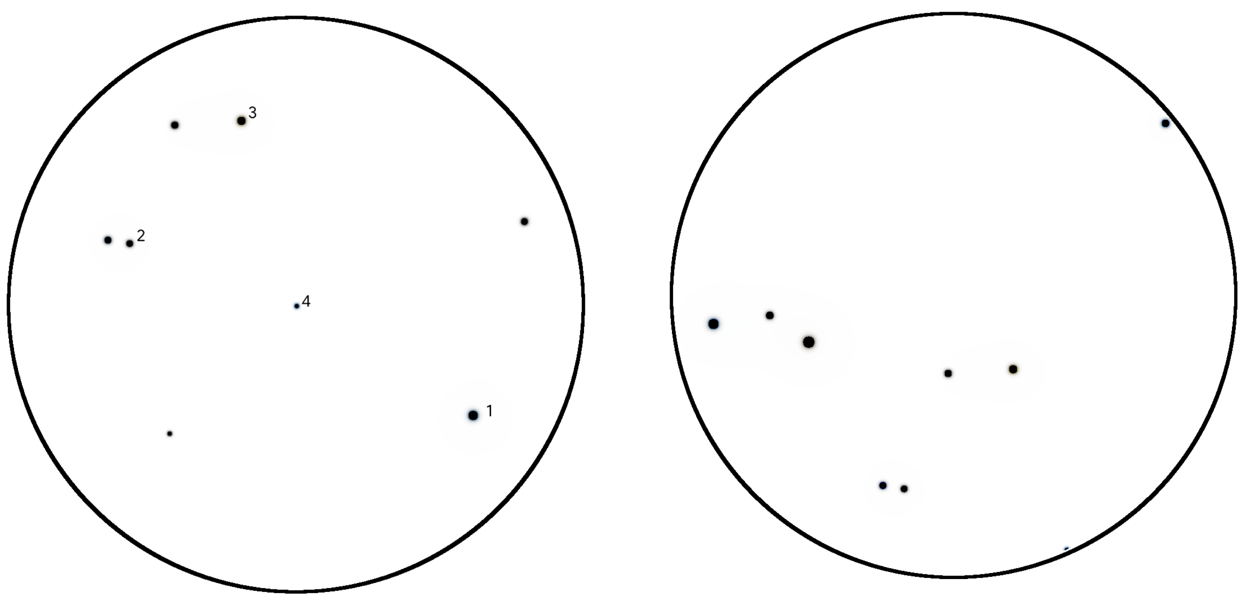
$$(\alpha, \delta) = (14^{\mathrm{h}}16^{\mathrm{m}}49.46^{\mathrm{s}}, -5^\circ 35' 15.7'').$$

However, Berwyn wants to find out more information and you, as one of his friends, want to help him. Please help him find:

(a) **(16 points)** Orientation (cardinal points) of the telescope *(mark and label them in the answer sheet)*,

(b) **(10 points)** Estimated magnitude of Star 3, and

(c) **(20 points)** Telescope's FoV in degrees, please answer in 3 significant figures!



(a) Initial FoV

(b) FoV ater 81 seconds

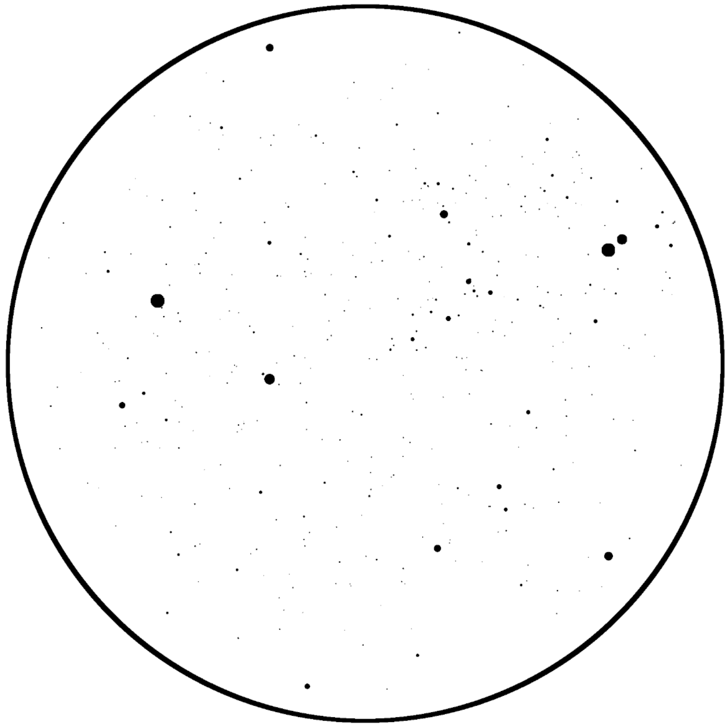
Figure 5: FoV of Berwyn's Telescope

4. **(54 points)** *Sky Map Attack 2!*

Please answer this question in the provided Question 4 Answer Sheet

After your first experience stargazing in a deserted island, you and your friends decided to go on another vacation to another island to practice your observation skills for IOAA 2026. One night, you looked up at the sky and it looked like the image below.

As the sky was bright, you and your friends gave each other quizzes about the sky.



(a) **(12 points)** List out 6 constellations above the Horizon.

(b) **(8 points)** Mark and label two out of three poles (North/South, whichever is above the Horizon): Celestial, Ecliptic, or Galactic (pick two).

(c) **(10 points)** What is the estimated latitude of the island? *Hint: the projection is azimuthal equidistant*

(d) **(8 points)** Mark and label the four cardinal points.

(e) **(8 points)** Draw and label two out of three planes: Celestial Equator, Ecliptic, or Galactic (pick two).

(f) **(3 points)** Are there any planets seen in the sky? If yes, mark and label the planets.

(g) **(5 points)** Mark and label: M4 or M13 (pick one).