2026 USAAAO Third Exam
(Data Analysis + Observation)

1 Instructions (Please Read Carefully)

The test must be completed within 2 hours (120 minutes) and the maximum number of points is **230 points**. The point breakdown per problem is as follows.

Problem	P1	P2	P3	P4	**Total**
Points	55	75	46	54	**230**

Please solve each problem on a blank piece of paper and mark the number of the problem at the top of the page. Some problems may require a designated answer sheet; these will be provided. The contestant's full name in capital letters should appear at the top of each solution page. If the contestant uses scratch papers, those should be labeled with the contestant's name as well and marked as "scratch paper" at the top of the page. Scratch paper will not be graded. Partial credit will be available given that correct and legible work was displayed in the solution.

This is a written exam. Contestants can only use a scientific calculator (non-programmable and non-graphing) for this exam. A table of physical constants will be provided. **Discussing the problems with other people is strictly prohibited in any way until the end of the examination period on June 25, 2026.** Receiving any external help during the exam is strictly prohibited. This means that the only allowed items are: a calculator, the provided table of constants, a pencil (or pen), an eraser, blank sheets of papers, and the exam. No books or notes are allowed during the exam. Students must be proctored during the entire duration of the exam.

This exam sheet and your answer sheets (including scratch papers) should be returned to your proctors once the exam ended. Your proctors should upload your answer sheets to the Google Form provided to them.

We acknowledge the following people for their contributions to this year's practical exam:

Ferdinand, Vincent Bian

After reading the instructions, please make sure to sign, affirming that:

1. All work on this exam has been completed by me.
2. I took this exam under the supervision of a proctor.
3. I did not receive any external help beyond the materials provided.
4. I will not discuss the contents of this exam with anyone until June 24, 2026.
5. Failure to follow these rules will result in disqualification from the exam.

Student Signature: ______________________________ Date: ______________________________

1. **(55 points)** *Type II Supernova*

In April 2024, the Asteroid Terrestrial-impact Last Alert System (ATLAS) reported the emergence of a rapid brightening object in the constellation Hydra. Follow-up observations confirmed that the transient was a nearby Type II supernova located at ~ 7.2 Mpc, one of the closest supernovae of the decade. Due to its proximity, the supernova, named SN 2024ggi, has been well-characterized in multiple wavelengths. It was revealed that the expanding supernova shock was moving outward at a nearly constant velocity of 10,000 km/s. Prior to the explosion, the progenitor star had been losing mass through a slow stellar wind with a velocity of 20 km/s, creating a dense circumstellar medium (CSM) around the star.

When the rapidly expanding shock collides with this surrounding material, the gas is heated to temperatures of millions of kelvin and emits strongly in X-rays. As such, X-ray observations are the best tracer to constrain supernovae properties. X-ray observations in 2024 revealed the supernova's unabsorbed/intrinsic fluxes and emission measure[1] (EM) and can be summarized in the table below.

Table 1: Observed unabsorbed flux and emission measure of the supernova

Time since explosion (days)	Unabsorbed Flux (erg s^{-1} cm^{-2})	EM (cm^{-3})
5.3	7.11×10^{-13}	2.75×10^{62}
10.9	4.98×10^{-13}	9.31×10^{61}
15.0	2.54×10^{-13}	6.08×10^{61}
16.2	2.85×10^{-13}	4.76×10^{61}
24.5	1.66×10^{-13}	3.76×10^{61}
34.8	1.31×10^{-13}	1.84×10^{62}
43.2	1.02×10^{-13}	1.07×10^{62}
55.0	0.75×10^{-13}	2.80×10^{61}
85.4	0.55×10^{-13}	2.16×10^{61}

In this problem, we are trying to find out the brightness decay rate of a supernova and calculate the mass-loss rate of the supernova progenitor through the shock interactions with the circumstellar medium (CSM). For this problem, please report the final answer using time in days and other parameters in cgs (centimeter-gram-seconds) units.

(a) **(20 points)** Plot the luminosity vs time (in the provided grid paper). The luminosity vs time has a power-law relation, i.e. $L = At^{\alpha}$, please estimate A and α to get the relation and draw the relation in the plot too. *Hint: it may be useful to draw a log-log plot.*

(b) **(23 points)** As the supernova explosion expands, the surrounding CSM of the supernova is being affected. Brethauer (2022) derived that the unshocked CSM density is expressed by:

$$\rho_{\mathrm{CSM}} = \frac{m_p}{4}\sqrt{\frac{2 \times \mathrm{EM} \times \mu_e \times \mu_I}{V_{\mathrm{FS}}}}$$

where m_p is the mass of proton, μ_e is the mean molecular weight per electron and can be assumed to be $\mu_e = 1.14$, μ_I is the mean molecular weight per ion and can be assumed to be $\mu_I = 1.24$, and $V_{\mathrm{FS}} \approx \frac{4}{3}\pi\left((1.2 \cdot R_{\mathrm{shock}})^3 - R^3_{\mathrm{shock}}\right)$ is the volume of the forward shock. Now, find the unshocked CSM densities and plot them as a function of shock radius (in the provided grid paper). The density vs radius also has a power-law relation, i.e. $\rho = Br^{\beta}$, please estimate B and β to get the relation and draw the relation in the plot too. *Hint: it may be useful to draw a log-log plot.*

(c) **(12 points)** Based on your previous results, please find the mass-loss rate and uncertainty of the supernova progenitor in $M_\odot$/yr. (*Hint: What is the relationship between density, radius, wind speed, and mass-loss rate? Disregard your fitting results in part b*)

[1] Emission measure (EM), usually used in X-ray and Extreme Ultraviolet astronomy, is the number density of electron times the number density of ion integrated over volume of the region. In other words: $\mathrm{EM} = \int n_e n_I \, \mathrm{d}V$.

Solution: This problem is about the supernova SN 2024ggi and inspired by one of my research projects (The data is from there too!). Please read it in `https://doi.org/10.3847/1538-4357/ae4ec6` for more background information or if you are interested in learning more about core-collapse supernova!

General plot rubric for this question:

Using less than 70% of the total area for the graph	Deduct 20% of the marks for correct graph
Non-constant scale for axis	Deduct 20% of the marks for correct graph
Missing or incorrect labels or units in the axes	Deduct 20% of the marks for correct graph

(a) The luminosity is calculated from the observed unabsorbed flux using

$$L = 4\pi D^2 F,$$

where $D = 7.2$ Mpc $= 7.2 \times 10^6$ pc $= 2.22 \times 10^{25}$ cm. **(3 points)** Calculating the luminosity for each observation, we can get **(5 points)**

t (days)	$\log_{10}(t)$	F (erg s^{-1} cm^{-2})	L (erg s^{-1})	$\log_{10}(L)$
5.3	0.724	7.11×10^{-13}	4.41×10^{39}	39.644
10.9	1.037	4.98×10^{-13}	3.09×10^{39}	39.490
15.0	1.176	2.54×10^{-13}	1.58×10^{39}	39.199
16.2	1.210	2.85×10^{-13}	1.77×10^{39}	39.248
24.5	1.389	1.66×10^{-13}	1.03×10^{39}	39.013
34.8	1.542	1.31×10^{-13}	8.13×10^{38}	38.910
43.2	1.635	1.02×10^{-13}	6.33×10^{38}	38.801
55.0	1.740	0.75×10^{-13}	4.65×10^{38}	38.667
85.4	1.931	0.55×10^{-13}	3.41×10^{38}	38.533

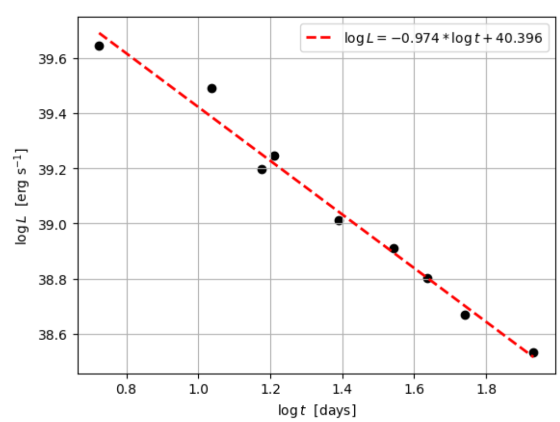
Then, fit the luminosity decay rate with power law:

$$L = At^{\alpha} \quad \leftrightarrow \quad \log L = \alpha \log t + C$$

We get $\alpha \approx -0.974$ and $C = 40.396$ or in other words:

$$\log L = -0.974 \log t + 40.396$$

(5 points) The plot: **(7 points)**



(b) The shock radius is

$$R_{\text{shock}} = v_{\text{shock}} t,$$

where $v_{\rm shock} = 10000$ km s^{-1} = 1.0×10^9 cm s^{-1}. **(3 points)** Using the fact that $\mathrm{EM} = \int n_e \cdot n_I \, dV$, the unshocked CSM density is

$$\rho_{\rm CSM} = \frac{m_p}{4} \sqrt{\frac{2 \times \mathrm{EM} \times \mu_e \times \mu_I}{V_{\rm FS}}}$$

where $m_p = 1.67 \times 10^{-24}$ g, $\mu_e = 1.14$, and$\mu_I = 1.24$. Calculating the unshocked CSM density, we can get **(7 points)**

t (days)	$R_{\rm shock}$ (cm)	$\log_{10}(R_{\rm shock})$	$\rho_{\rm CSM}$ (g cm^{-3})	$\log_{10}(\rho_{\rm CSM})$
5.3	4.58×10^{14}	14.661	6.81×10^{-16}	-15.167
10.9	9.42×10^{14}	14.974	1.34×10^{-16}	-15.873
15.0	1.30×10^{15}	15.114	6.73×10^{-17}	-16.172
16.2	1.40×10^{15}	15.146	5.30×10^{-17}	-16.276
24.5	2.12×10^{15}	15.326	2.54×10^{-17}	-16.595
34.8	3.01×10^{15}	15.479	3.31×10^{-17}	-16.480
43.2	3.73×10^{15}	15.572	1.83×10^{-17}	-16.738
55.0	4.75×10^{15}	15.677	6.50×10^{-18}	-17.187
85.4	7.38×10^{15}	15.868	2.95×10^{-18}	-17.530

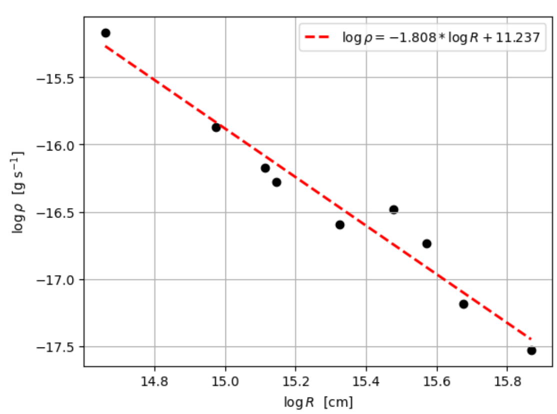
Then, fit the density and shocked radius with a power law,

$$\rho_{\rm CSM} = B R_{\rm sh}^{\beta} \quad \leftrightarrow \quad \log \rho_{\rm CSM} = \beta \log R_{\rm sh} + K$$

We get $\beta \approx -1.808$ and $K = 11.237$ or in other words:

$$\log \rho_{\rm CSM} = -1.808 \log R_{\rm sh} + 11.237$$

(5 points) The plot: **(8 points)**



(c) For a steady stellar wind,

$$\rho_{\rm CSM} = \frac{\dot{M}}{4\pi R^2 v_{\rm wind}} \quad \leftrightarrow \quad \dot{M} = 4\pi R^2 \rho_{\rm CSM} v_{\rm wind}.$$

(3 points) This is why your β value in part b should be near -2. However, since our data was not perfect, we should disregard our fitting results from part b. Using $v_{\rm wind} = 20$ km s^{-1} = 2×10^6 cm s^{-1}, we can get the mass loss rate: **(5 points)**

R_{shock} (cm)	ρ_{CSM} (g cm^{-3})	$\dot{M}$ ($M_\odot$ yr^{-1})
4.58×10^{14}	6.81×10^{-16}	5.69×10^{-5}
9.42×10^{14}	1.34×10^{-16}	4.75×10^{-5}
1.30×10^{15}	6.73×10^{-17}	4.52×10^{-5}
1.40×10^{15}	5.30×10^{-17}	4.15×10^{-5}
2.12×10^{15}	2.54×10^{-17}	4.54×10^{-5}
3.01×10^{15}	3.31×10^{-17}	1.20×10^{-4}
3.73×10^{15}	1.83×10^{-17}	1.01×10^{-4}
4.75×10^{15}	6.50×10^{-18}	5.86×10^{-5}
7.38×10^{15}	2.95×10^{-18}	6.41×10^{-5}

and the mean value **(2 points)** and the uncertainty (i.e. standard deviation) **(2 points)** is

$$\boxed{\dot{M} \approx (6.4 \pm 2.6) \times 10^{-5}\ M_\odot\ \text{yr}^{-1}.}$$

Using sample standard deviation is also okay, getting an error of $\sim 2.8 \times 10^{-5}\ M_\odot\ \text{yr}^{-1}$

2. **(75 points)** *Travelers in Outer Space*

Esker, Chert, and Gabbro are traveling in a spaceship in the Solar System when their spaceship explodes! Fortunately, everyone makes it to an escape pod in time and escapes safely, but they end up all scattered in different locations.

Note: For all questions asking you to compute a numerical answer from the data provided, include the uncertainty in your answer.

(a) (25 points total) Esker's escape pod ends up in orbit around some planet in the Solar System. They gaze longingly at Earth through a telescope, and every day they measure the apparent angular diameter of the Earth. Unfortunately, they don't have any precise instruments, so they can only measure the *relative* angular size of the Earth. You may assume that the absolute angular size of Earth is small.

The following is a table of the relative size over time. *The uncertainty in each measurement is 0.05.*

Table 2: Apparent relative angular diameter of Earth as seen from Esker's pod (arbitrary units)

Day	**Rel. size**	**Day**	**Rel. size**	**Day**	**Rel. size**
0	0.87	440	3.99	930	1.46
63	0.78	462	3.26	989	2.68
112	0.59	478	2.57	1004	3.34
169	0.77	543	1.28	1030	4.02
219	0.72	592	0.96	1045	3.54
284	0.87	653	0.96	1062	2.74
325	1.04	700	0.64	1079	2.12
392	2.35	756	0.62	1144	1.02
402	2.58	824	0.69	1190	0.87
418	3.33	869	0.98	1249	0.73

(i) **(5 points)** Let d_{near} be the closest distance between Earth and Esker, and let d_{far} be the farthest distance. What is the ratio $\frac{d_{\text{far}}}{d_{\text{near}}}$?

(ii) **(10 points)** What is the period of Esker's motion relative to the Earth?

(iii) **(10 points)** What is the radius of Esker's orbit around the sun? Where is Esker?

(b) (25 points total) Chert's escape pod lands on a rocky body in the Solar System, and they can't find Earth! Fortunately, they have a very accurate telescope, a star-chart computer, and a gyroscope that locks their coordinate frame to Earth's equatorial axes.

Chert measures the RA and declination of many bright stars and compares each measurement to the catalog value from Earth at the same date. The resulting angular offsets measured in *milli-arcseconds*, caused by the parallax of Chert's location relative to Earth, are listed below in Table 3 along with each star's distance. The uncertainty in each measurement is 0.001 mas (it is a *very* precise telescope).

Table 3: Chert's stellar parallax measurements

Star	**Day**	θ **(mas)**	D **(ly)**	**Star**	**Day**	θ **(mas)**	D **(ly)**
Aldebaran	0	0.123	65	Dschubba	15	0.020	402
Tianguan	2	0.020	417	Tania	15	0.024	249
Furud	2	0.025	336	Chertan	16	0.020	178
Mirach	3	0.019	199	Shaula	17	0.009	703
Ankaa	4	0.096	77	Ascella	17	0.098	89
Fawaris	4	0.046	171	Seginus	18	0.081	85
Scheat	4	0.030	199	Hadar	22	0.013	525
Fulu	5	0.011	597	Porrima	22	0.205	39
Menkalinan	5	0.066	82	Izar	22	0.038	210
Alkaphrah	7	0.015	423	Unukalhai	22	0.074	73
Zosma	8	0.139	58	Matar	23	0.037	215
Meissa	9	0.005	1056	Sargas	23	0.010	272
Aludra	9	0.003	3198	Kitalpha	24	0.030	186
Gomeisa	11	0.022	170	Dalim	25	0.172	46
Cor	12	0.059	110	Azha	26	0.058	133

Ignore aberration caused by orbital motion.

(i) **(5 points)** Suppose Chert is a distance r from Earth, and measuring the apparent location of a star which is a distance D from Earth (you may assume $D \gg r$). What are the possible values Chert might get for the angular distance θ between the apparent positions for that star on Earth vs at Chert's location?

(ii) **(12 points)** Plot θ vs $\frac{1}{D}$. Using this plot, determine how far Chert is from the Earth.

(iii) **(3 points)** Why do more of the data points lie closer to the maximum possible angular displacement than the minimum possible angular displacement?

(iv) **(5 points)** Where is Chert, and why might Earth not be visible from their location?

(c) **(25 points)** Gabbro's escape pod somehow ends up in orbit around another star! Just like Chert, they have a telescope, a star-chart computer, and a gyroscope that locks their coordinate frame to Earth's equatorial axes.

They measure the right ascension and declination of a few known stars and compare it to the equatorial coordinates of those stars when observed from Earth.

Table 4: Apparent positions of stars as seen from Earth and from Gabbro's pod

Star	**RA (Earth)**	**Dec (Earth)**	**RA (Gabbro)**	**Dec (Gabbro)**	**Dist (ly)**
Nekkar	$15^{h}01.95^{m}$	$+40^\circ\,23.44'$	$15^{h}21.41^{m}$	$+30^\circ\,09.67'$	219
Dubhe	$11^{h}03.73^{m}$	$+61^\circ\,45.07'$	$12^{h}52.67^{m}$	$+50^\circ\,51.11'$	124
Tania Australis	$10^{h}17.10^{m}$	$+42^\circ\,54.87'$	$11^{h}26.35^{m}$	$+32^\circ\,42.87'$	134
Ruchbah	$01^{h}25.81^{m}$	$+60^\circ\,14.12'$	$23^{h}00.46^{m}$	$+53^\circ\,12.78'$	99
Capella	$05^{h}16.69^{m}$	$+45^\circ\,59.94'$	$06^{h}29.42^{m}$	$+39^\circ\,47.67'$	43
Muscida	$08^{h}30.27^{m}$	$+60^\circ\,43.11'$	$09^{h}39.48^{m}$	$+60^\circ\,25.18'$	184
Polaris	$02^{h}31.78^{m}$	$+89^\circ\,15.85'$	$17^{h}49.74^{m}$	$+86^\circ\,22.46'$	431
Tania Borealis	$10^{h}22.33^{m}$	$+41^\circ\,29.97'$	$10^{h}58.42^{m}$	$+36^\circ\,30.60'$	249
Algol	$03^{h}08.17^{m}$	$+40^\circ\,57.34'$	$01^{h}52.16^{m}$	$+32^\circ\,28.99'$	93
Menkalinan	$05^{h}59.53^{m}$	$+44^\circ\,56.85'$	$06^{h}41.64^{m}$	$+42^\circ\,50.87'$	82
Mirach	$01^{h}09.73^{m}$	$+35^\circ\,37.25'$	$00^{h}30.19^{m}$	$+29^\circ\,41.37'$	199
Grumium	$17^{h}53.53^{m}$	$+56^\circ\,52.35'$	$17^{h}41.57^{m}$	$+34^\circ\,59.80'$	111
Schedar	$00^{h}40.51^{m}$	$+56^\circ\,32.24'$	$23^{h}47.19^{m}$	$+53^\circ\,28.47'$	229

Where is Gabbro?
You are provided with an answer sheet with a gnomonic projection of the celestial sphere.
Hint: lines in a gnomonic projection correspond to great circles on the sphere

Solution:

(a) (i) **(5 points)** To get a sense of the motion, we need to make a plot such as the one in Figure 1, and draw a trend line. Note that the line in Figure 1 shows the *true* value, based on real astronomical data.

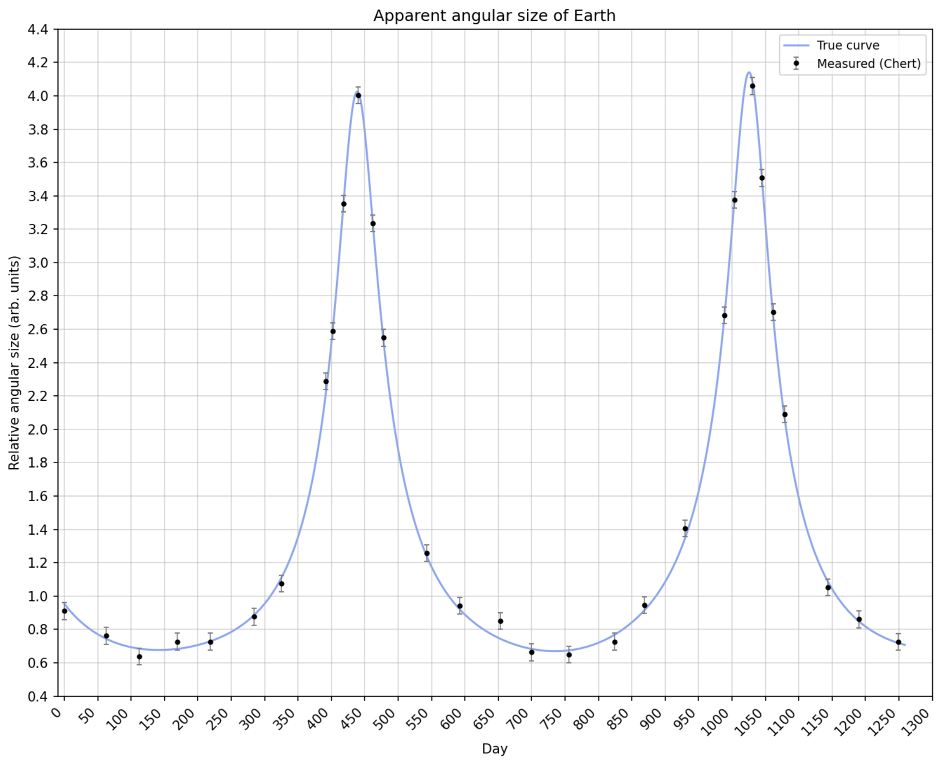


Figure 1: Plot of Chert's observations.

From drawing a best fit line, the peak seems to be around 4.1 ± 0.1 and the lowest looks to be around 0.6 ± 0.05.

This gives a ratio of $\frac{4.1}{0.6} = 6.8$. The relative uncertainty in the quotient is $\sqrt{\left(\frac{0.1}{4.1}\right)^2 + \left(\frac{0.05}{0.6}\right)^2} = 0.09$, meaning that the ratio with uncertainty would be $\boxed{6.8 \pm 0.6}$. This is also the answer one arrives at by reading off the largest and smallest values from the table, which is also acceptable for this part.

Any uncertainty in the range 0.4 to 0.12 would be acceptable, as determining the uncertainty from the graph would be subjective.

(ii) **(10 points)**
From the plot, we see that the peaks on days 440 and 1030 are close to the true peaks. We can estimate the period to be $1030 - 440 = \boxed{590 \pm 30}$ days (any uncertainty from 10 to 50 days is reasonable).

(iii) **(10 points)** We first consider the result of part (a)(i). Suppose Chert is orbiting at a distance of x AU from the sun. The farthest distance from the Earth is $1 + x$, and the nearest distance from the Earth is either $x - 1$ or $1 - x$, depending on whether $x > 1$.

If we assume $x < 1$, the equation becomes $\frac{1+x}{1-x} = 6.8 \pm 1.2$, (using the upper bound of the range of uncertainty) which gives $x = 0.74 \pm 0.05$. If $x > 1$, the equation is $\frac{1+x}{x-1} = 6.8$, which gives $x = 1.34 \pm 0.10$. The uncertainties in this part of the solution were computed by solving the equation using the upper and lower bounds of 6.8 ± 1.2.

From (a)(ii), the synodic period of Chert's orbit with Earth is 590 ± 50 days (using the upper range of acceptable uncertainties). If the sidereal period of Chert's orbit is T days, then either $\frac{1}{T} - \frac{1}{365} = \frac{1}{590}$, or $\frac{1}{365} - \frac{1}{T} = \frac{1}{590}$, depending on whether $T > 365$.

Solving these equations for T, we get that T is either 960 ± 80 days or 225 ± 8 days. By Kepler's 3rd law, these correspond to orbits of radius $\left(\frac{960 \pm 80}{365}\right)^{2/3} = 1.9 \pm 0.2$ AU and $\left(\frac{225 \pm 8}{365}\right)^{2/3} = 0.72 \pm 0.02$ AU.

The only orbital radius consistent with both parts would be about 0.73 AU, which corresponds to the orbit of $\boxed{\text{Venus}}$.

(b) (i) **(5 points)** Let the location of the Earth be E, the location of the star be S, and the location of Esker be K.

The angular displacement observed by Esker is equal to the measure of $\angle ESK$. Using the law of sines, $\frac{EK}{\sin \angle ESK} = \frac{SK}{\sin \angle SEK}$.

We then use the following:

- $EK = r$ (from the problem statement)
- $\sin \angle ESK = \angle ESK$ (using a small angle approximation)
- $SK = D$ (because $D \gg r$)

Then, we get $\angle ESK = \frac{r}{D} \sin SEK$.

Because $\sin \angle SEK$ ranges from 0 to 1, we get that $\boxed{0 \leqslant \theta \leqslant \frac{r}{D}}$.

(ii) **(12 points)** If we plot the angular displacement θ vs $1/D$, we have from part (b)(i) that all the points should lie below a line of slope r. If we make this plot and draw a line as in Figure 2, the points seem to lie below a line of slope $r = 8.7$ mas/ly^{-1}.

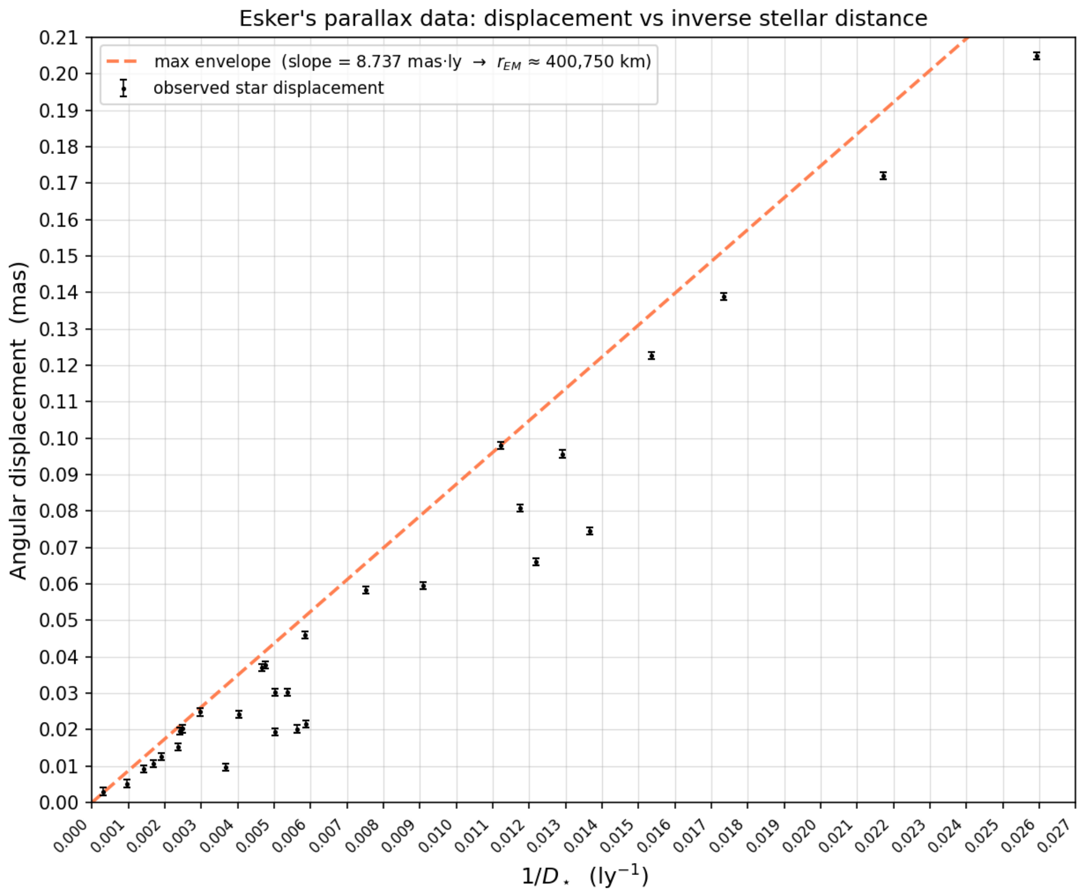


Figure 2: Plot of θ vs $1/D$ from Esker's observation.

Some uncertainty comes from the chance that there aren't any stars with the exact maximum parallax, so any estimate of the uncertainty from 0.1 to 0.5 is reasonable.

Converting mas into radians, we get that

$$r = (8.7 \pm 0.3)\ \text{mas/ly}^{-1} = 4.2 \pm 0.1 \times 10^{-8}\ \text{ly} = \boxed{4.0 \pm 0.1 \times 10^{8}\ \text{m}}.$$

(iii) **(3 points)** As in part (b)(i), let the location of the Earth be E, the location of the star be S, and the location of Esker be K. Suppose that $\overrightarrow{ES}$ is a random direction on the celestial sphere. $\angle SEK$ is smaller when $\overrightarrow{ES}$ is closer to the poles. Since more of the area of a sphere lies away from the poles, larger values of $\angle SEK$, and thus larger values of θ relative to the maximum value, are more likely. Figure 3 shows a visualization of this effect.

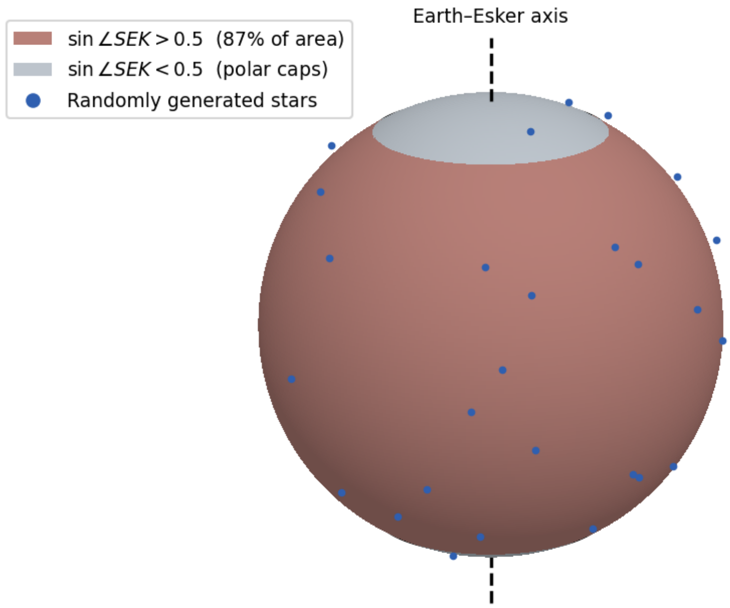


Figure 3: Plot of Chert's observations.

Any explanation mentioning *either* the fact that $\angle SEK$ is likely to be close to $90°$, *or* that the distribution of $\sin\theta$ for $\theta \in [0°, 180°]$ is biased towards larger values, should get full credit.

(iv) **(5 points)** From part (b)(ii), Esker is around 4×10^8 m from the Earth, which should be recognizable as being around the Earth-Moon distance.

Since the Moon is tidally locked to the Earth, the reason Esker never sees Earth could be that they are on $\boxed{\text{the far side of the Moon}}$.

(c) **(25 points)** Let G be the (unknown) apparent location of Gabbro on the celestial sphere as viewed from Earth. For a given star, let S be the apparent location of that star from Earth, and S' be the apparent location of that star from Gabbro's location. The key insight is that G, S, and S' all lie on a great circle.

One way to prove this assertion is to consider the locations of Earth, Gabbro, and the star in 3D space. Let O be the position of the Earth in 3D space, let X be Gabbros' position, and let Y be the star's location.

Then, the direction of G on the celestial sphere is the direction of the vector $\overrightarrow{OX}$. Similarly, S corresponds to the direction of $\overrightarrow{OY}$, and S' corresponds to $\overrightarrow{XY}$. Note that all three of these vectors lie within the plane passing through O, X, and Y. This means that the direction of those vectors lie on the plane passing through the origin, parallel to this plane. Since the intersection of a sphere and a plane passing through its center is a great circle, this concludes the proof.

Thus, we can plot the position of a few stars from Earth's perspective and from Gabbro's perspective on a gnomonic projection, draw the line connecting them, and see where they all intersect. This is shown in Figure 4.

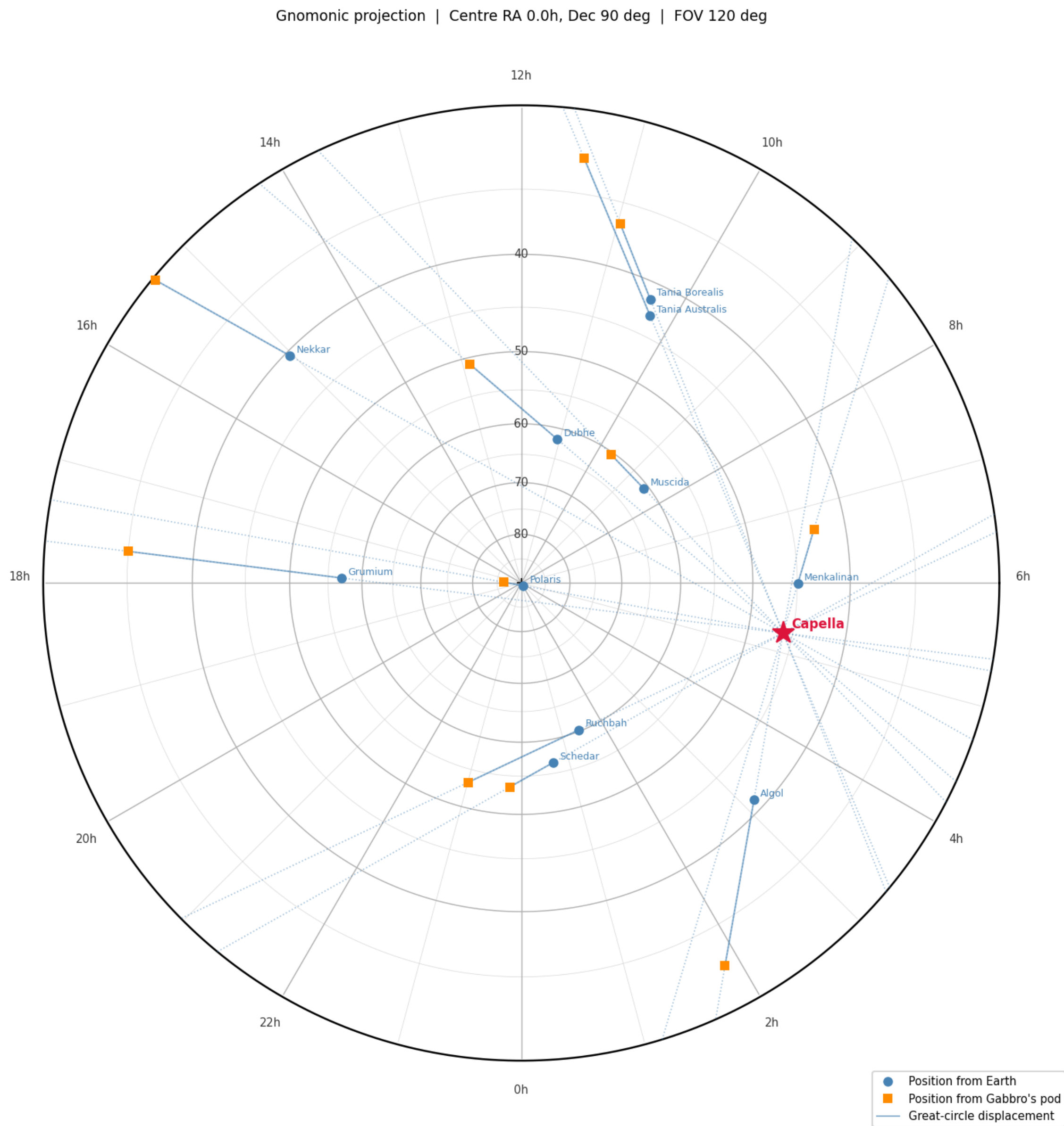


Figure 4: Plot of apparent location from Earth vs Gabbro's location.

From the figure, we see that all the lines intersect around $\boxed{\text{Capella}}$, whose location is also listed in the table.

3. **(46 points)** *Stars in the Telescope*

Please answer this question in the provided Question 3 Answer Sheet

Berwyn, an amateur astronomer, is observing at a random star in the night sky through his telescope. He first positioned the star in the center of his telescope's field of view (FoV) and labeled some of the stars as seen in Figure 5a. Since the telescope is not tracking the sky, the star slowly drifts across the telescope's FoV due to Earth's rotation. After 81.0 seconds, the telescope FoV looks like Figure 5b.

Berwyn then consults with his friends on his observations and gains additional information:

- Star 1 has magnitude 7.70.
- Star 2 has magnitude 9.8.
- Star 4 has magnitude 10.25 and coordinates

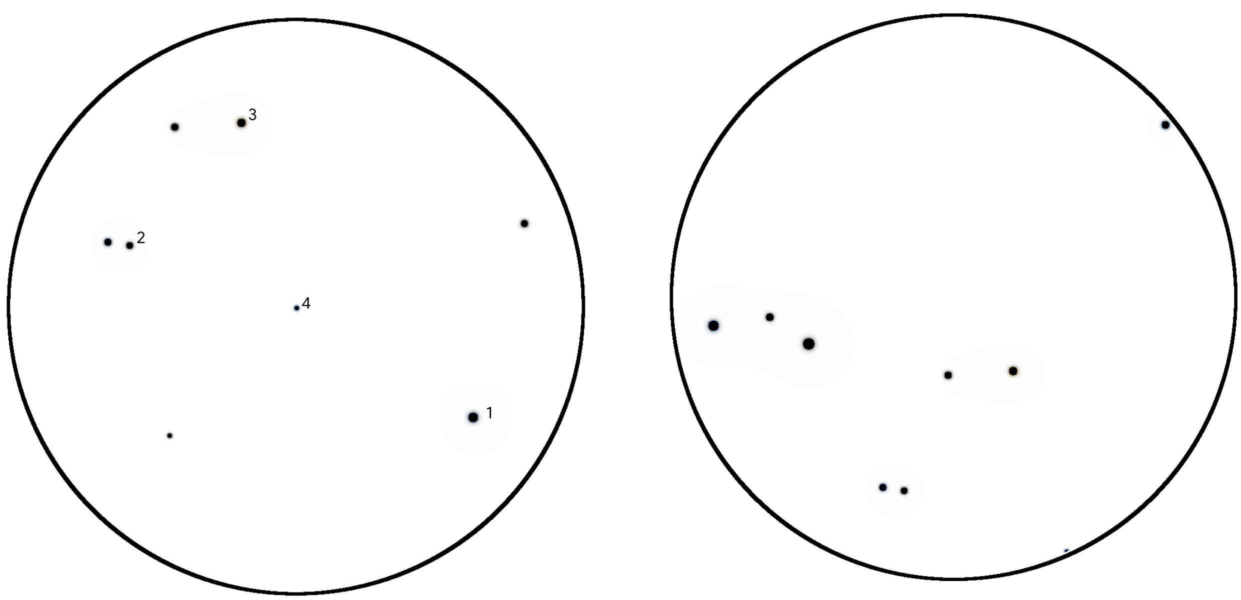
$$(\alpha, \delta) = (14^{\mathrm{h}}16^{\mathrm{m}}49.46^{\mathrm{s}}, -5°35'15.7'').$$

However, Berwyn wants to find out more information and you, as one of his friends, want to help him. Please help him find:

(a) **(16 points)** Orientation (cardinal points) of the telescope *(mark and label them in the answer sheet)*,

(b) **(10 points)** Estimated magnitude of Star 3, and

(c) **(20 points)** Telescope's FoV in degrees, please answer in 3 significant figures!



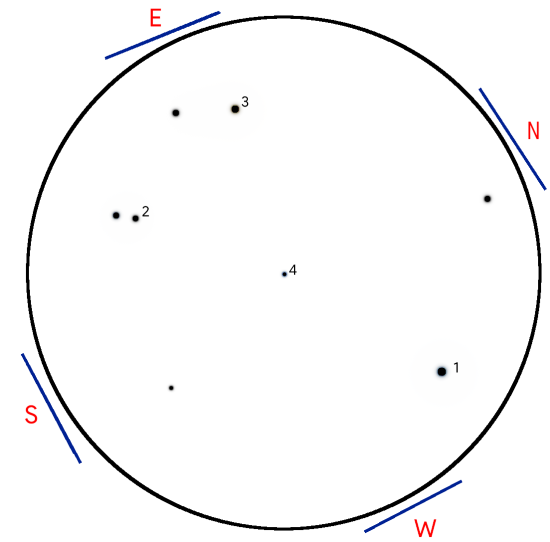
(a) Initial FoV

(b) FoV ater 81 seconds

Figure 5: FoV of Berwyn's Telescope

Solution:

(a) Stars always go from East to West, so the direction Star 2, 3, and 4 heading is West. In a sky map, the north is always 90 degrees clockwise of the east. More specifically, the orientation can be seen below. Each correct orientation is 4 points. Give full points if the cardinal point is within the blue line.



(b) An educated guess from the size of Star 1, Star 2, and Star 3, Star 3's magnitude is around 8.40. Answer between 8.20 and 8.60 is full 10 points, answer between 8.00 and 8.20 and between 8.60 and 8.80 is 5 points, else is 0.

(c) Star 4 essentially drifts from the center of the FoV to the edge of the FoV. This means Star 4 traverses half the FoV in 81 seconds. Hence, the whole FoV is:

$$\text{FoV} = \frac{360^\circ \times \cos\delta_{\text{Star 4}}}{23^{\text{h}}56^{\text{m}}4^{\text{s}}} \times 81\text{ s} \times 2 \approx 0.6736^\circ$$

Rubric:

- -2 point for using units other than degrees
- -3 point for using 24 hours instead of $23^{\text{h}}56^{\text{m}}4^{\text{s}}$
- -3 point for not using $\cos\delta_{\text{Star 4}}$
- -8 points for not multiplying by 2

4. **(54 points)** *Sky Map Attack 2!*

Please answer this question in the provided Question 4 Answer Sheet

After your first experience stargazing in a deserted island, you and your friends decided to go on another vacation to another island to practice your observation skills for IOAA 2026. One night, you looked up at the sky and it looked like the image below.

As the sky was bright, you and your friends gave each other quizzes about the sky.

(a) **(12 points)** List out 6 constellations above the Horizon.

(b) **(8 points)** Mark and label two out of three poles (North/South, whichever is above the Horizon): Celestial, Ecliptic, or Galactic (pick two).

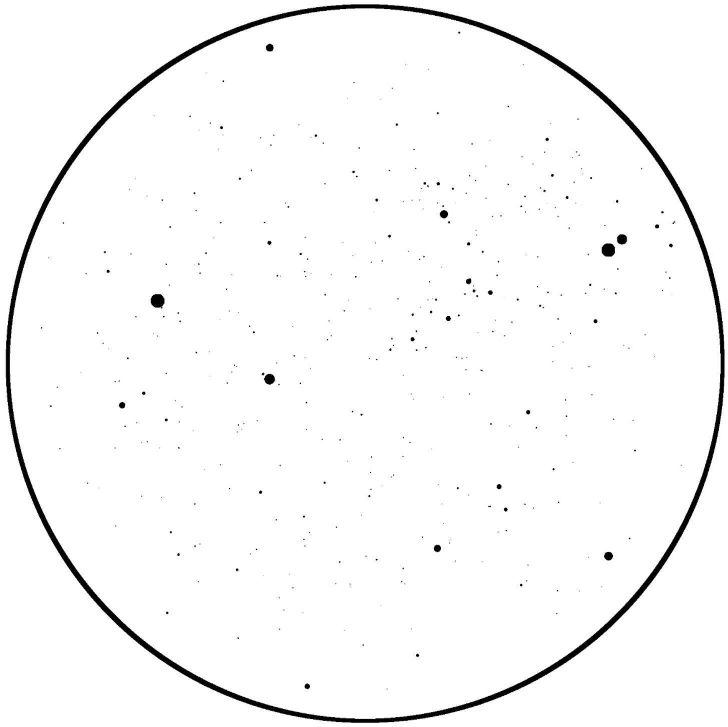
(c) **(10 points)** What is the estimated latitude of the island? *Hint: the projection is azimuthal equidistant*

(d) **(8 points)** Mark and label the four cardinal points.

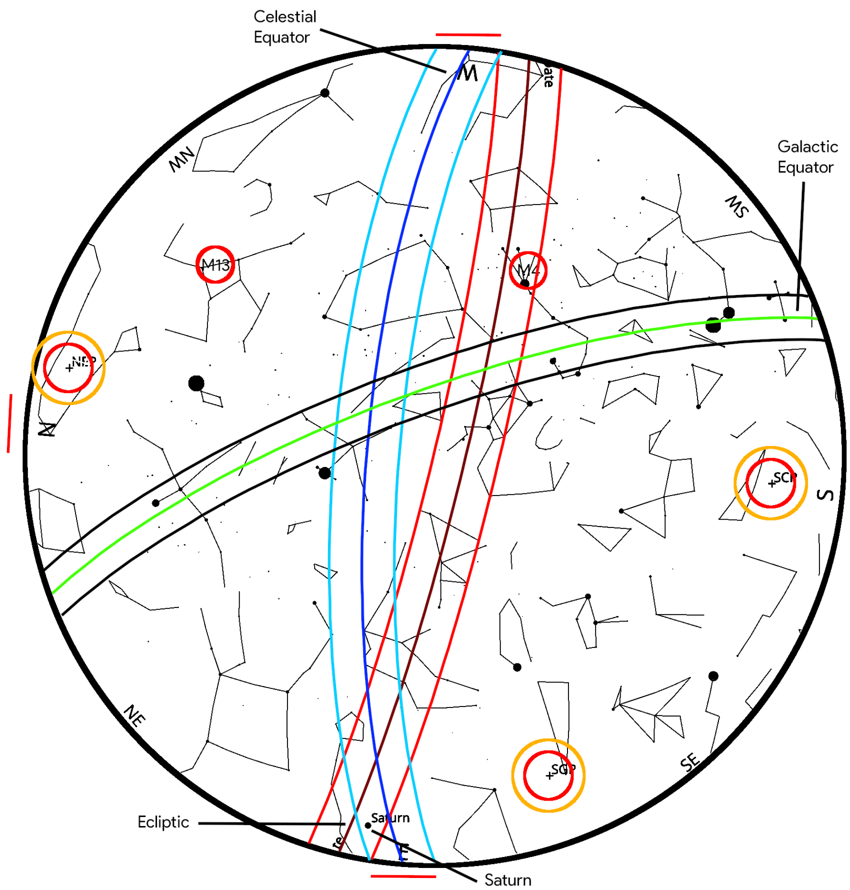
(e) **(8 points)** Draw and label two out of three planes: Celestial Equator, Ecliptic, or Galactic (pick two).

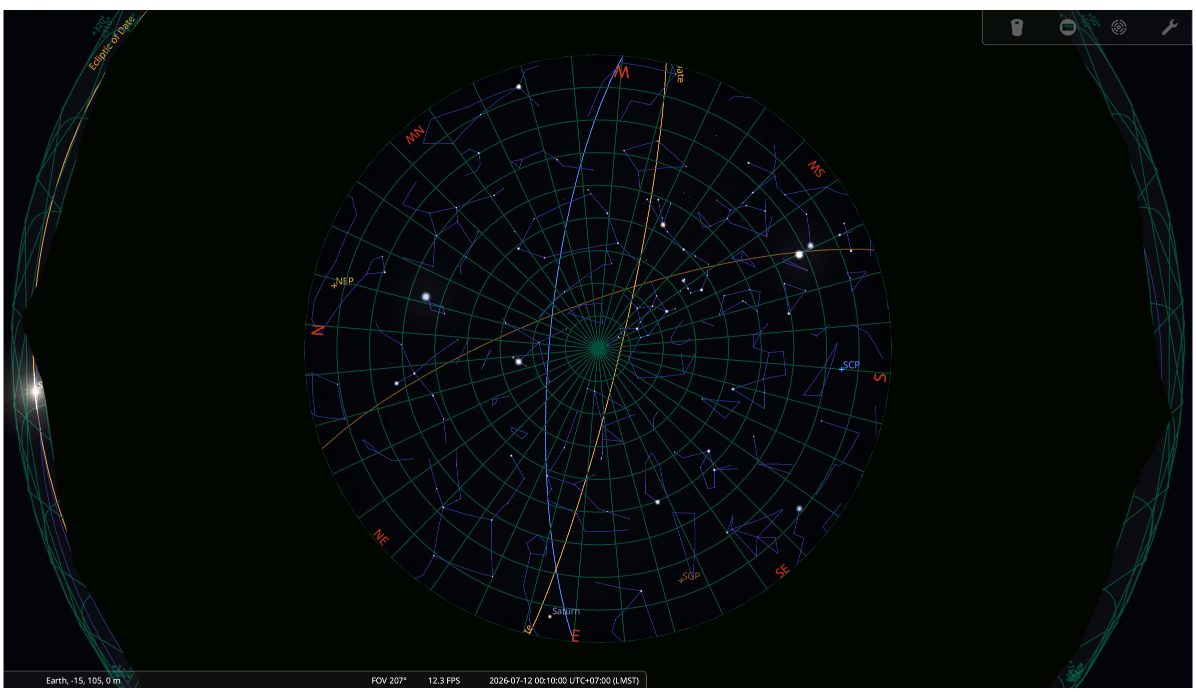
(f) **(3 points)** Are there any planets seen in the sky? If yes, mark and label the planets.

(g) **(5 points)** Mark and label: M4 or M13 (pick one).



Solution: Please refer to these two images below for the sky map. The first image is the inverted image, while the second image is the screenshot from Stellarium.





(a) Each correct constellation is 2 points. Available constellations are: *Draco, Lacerta, Pisces, Andromeda, Pegasus, Lyra, Cygnus, Hercules, Cepheus, Corona Borealis, Bootes, Serpens, Ophiuchus, Vulpecula, Sagitta, Delphinus, Scutum, Equuleus, Aquila, Aquarius, Capricornus, Virgo,* *Libra, Scorpius, Sagittarius, Crux, Corona Australis, Centaurus, Apus, Cetus, Piscis Austrinus, Grus, Phoenix, Sculptor, Tucana, Indus, Octans, Lupus, Norma, Ara, Pavo, Musca, Chamaleon, Hydrus, Triangulum Australe, Eridanus*

(b) Each pole is 4 points. Minus 1 point for mark correctly but labeled wrong. If marked within the red circle, it's full 4 points. If marked within the orange circle, it's 2 points. Outside is 0 points.

(c) In the sky, we can see Rigil Kentaurus and Crux constellation. We can make a rough isosceles triangle with Rigil Kent and Acrux as the base and the side is around 4 times the length of Acrux and Gacrux. We can estimate that the South Celestial Pole (SCP) altitude is around $a_{\text{SCP}} = 1$ cm and the distance to Zenith is around $R = 6$ cm. As it is azimuthal equidistant, we can get the latitude of the island:

$$\varphi = a_{\text{SCP}} = \frac{1}{6} \times 90^\circ = 15^\circ \text{ S}$$

In this subpart, 11° S $- 20^\circ$ S gets full 10 points; 5° S $- 10^\circ$ S or 10° S $- 25^\circ$ S gets 4 points; and outside that range gets no points. If you didn't explain your thoughts, you'd only get 2.5 points if it's within 10° S $- 20^\circ$ S and no points outside that.

(d) Each cardinal point is 2 points. Full points within the red line.

(e) Give full points if drawn and labeled correctly within the lines (90%-100% within). Half of the correct points if partially (50%-90%) within the lines. Zero otherwise

(f) There is only Saturn near the East cardinal point. If the answer is yes, but not labeled planet as Saturn, give 2 points. If the answer is yes, but Saturn/label is not marked correctly, give 1 point. No points given to labeling Saturn in a wrong star.

(g) Give full points as long as it's within the red circles.