USAAAO 2026 - First Round

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1. A planet moves in a circular orbit in a fixed plane with angular velocity ω, and let $\hat{n}$ be a unit vector perpendicular to the orbital plane. Let the planet's velocity vector as a function of time be given by

$$\vec{v}(t) = v_0\big(\cos(\omega t)\,\hat{i} + \sin(\omega t)\,\hat{j}\big)$$

Define $\vec{a}(t) = \frac{d\vec{v}}{dt}$. Which of the following statements is **correct**? *(Here, $\hat{i}$ and $\hat{j}$ are orthogonal unit vectors in the orbital plane.)*

(a) $\vec{a}$ is always parallel to $\vec{v}$

(b) $\vec{a}$ has a magnitude $v_0\omega$ and is perpendicular to $\vec{v}$

(c) $\vec{a}$ is zero since the magnitude of $\vec{v}$ is constant

(d) $\vec{v} \cdot \vec{a} = v_0^2\omega$

(e) $\vec{a}$ is parallel to $\hat{n}$

2. A planet of mass m orbits a star of mass M in an elliptical orbit with semi-major axis a and eccentricity e where $M \gg m$. Which of the following statements is **true**?

(a) The orbital speed is maximized at aphelion due to conservation of energy

(b) The planet moves in a perfect circle around the star regardless of eccentricity

(c) The orbital period depends on eccentricity through Kepler's Third Law

(d) The Roche limit increases if the planet's density decreases

(e) All five Lagrange points correspond to stable equilibria

3. Astronomers in the United States observe a spiral galaxy and measure its rotation curve. They find that beyond a radius r_0, the orbital speed of stars remains approximately constant at v_0. Which of the following conclusions is **most accurate**?

(a) The mass density of the galaxy must decrease faster than $1/r^2$

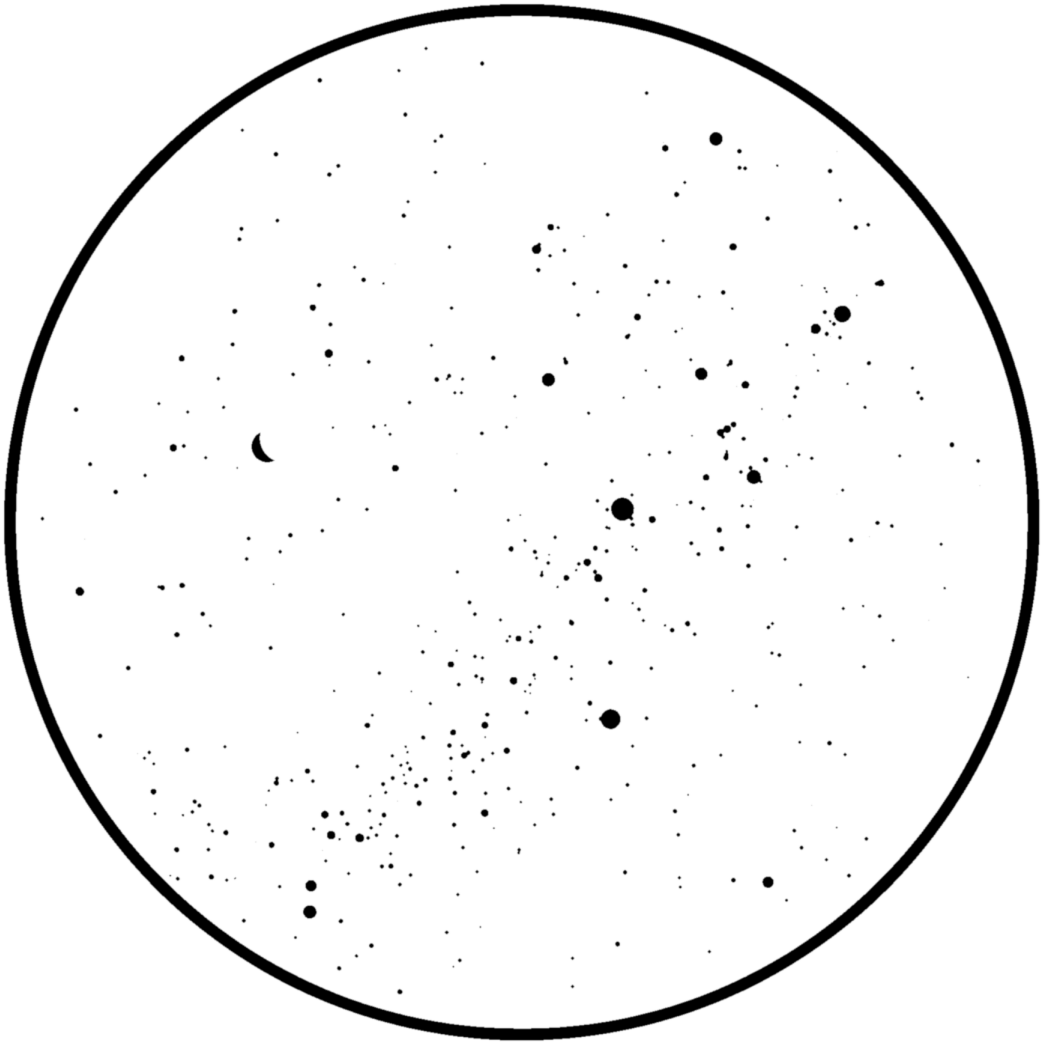
(b) The total enclosed mass becomes constant for $r > r_0$

(c) The luminosity profile directly traces the mass distribution

(d) The enclosed mass within radius r grows approximately linearly with r

(e) The galaxy cannot contain a central supermassive black hole

The following figure is used for questions 4, 5, and 6.



You are stranded on a remote island somewhere on Earth, at an unknown date and time (in the 21st century). You look up and see the night sky.

(For avoidance of doubt, the illuminated side of the Moon is on the left.)

4. Which of the following is the best estimate of your latitude?

   (a) 40° N
   (b) 20° N
   (c) 0°
   (d) 20° S
   (e) 40° S

5. Which of the following is the best estimate of the month of the year?

   (a) January
   (b) April

(c) July

(d) September

(e) November

6. Which of the following is the best estimate of the time of day (relative to local solar noon)?

   (a) 8:00 PM

   (b) 10:00 PM

   (c) 12:00 AM

   (d) 2:00 AM

   (e) 4:00 AM

Use the following information for Questions 7 and 8. One of Stephen Hawking's most famous predictions was the existence of Hawking radiation. Specifically, black holes act like thermal blackbodies and thus must emit blackbody radiation. According to Hawking, a black hole radiates a blackbody spectrum with peak wavelength λ_{max} proportional to its Schwarzschild radius R_s. Treat the surface of the blackbody as being at the black hole's event horizon.

7. According to the laws of thermodynamics and blackbody radiation, under this model, how does the power P radiated by a black hole scale with its mass M?

   (a) $P \propto M^{-5}$

   (b) $P \propto M^{-4}$

   (c) $P \propto M^{-3}$

   (d) $P \propto M^{-2}$

   (e) $P \propto M^{-1}$

8. Next, interpret the radiated power P from the previous problem as a gradual loss of the black hole's intrinsic mass-energy $E = Mc^2$. Consider the graph of the black hole's mass versus time. Which of the following best describes the shape of the resulting graph?

   (a) Increasing and concave up

   (b) Constant

   (c) Decreasing and concave down

   (d) Linearly decreasing

   (e) Decreasing and concave up

9. The asteroid Pallas orbits the Sun with a semi-major axis of 2.77 AU. What is its orbital period around the Sun in years?

   (a) $(2.77)^{1/2}$

   (b) $(2.77)^{3/2}$

   (c) $(2.77)^{5/2}$

   (d) $(2.77)^{2/3}$

   (e) $(2.77)^{2/5}$

10. A small satellite orbits the Earth so it stays directly above a fixed point on the equator. Using 5.97×10^{24} kg for Earth's mass and 6.37×10^{6} m for its radius, how far is the satellite from the surface of the Earth? Assume its mass is 300 kg and an orbital period of 86164 s. (Find the distance above the Earth's surface, not orbital radius.)

    (a) 3.58×10^{6} m

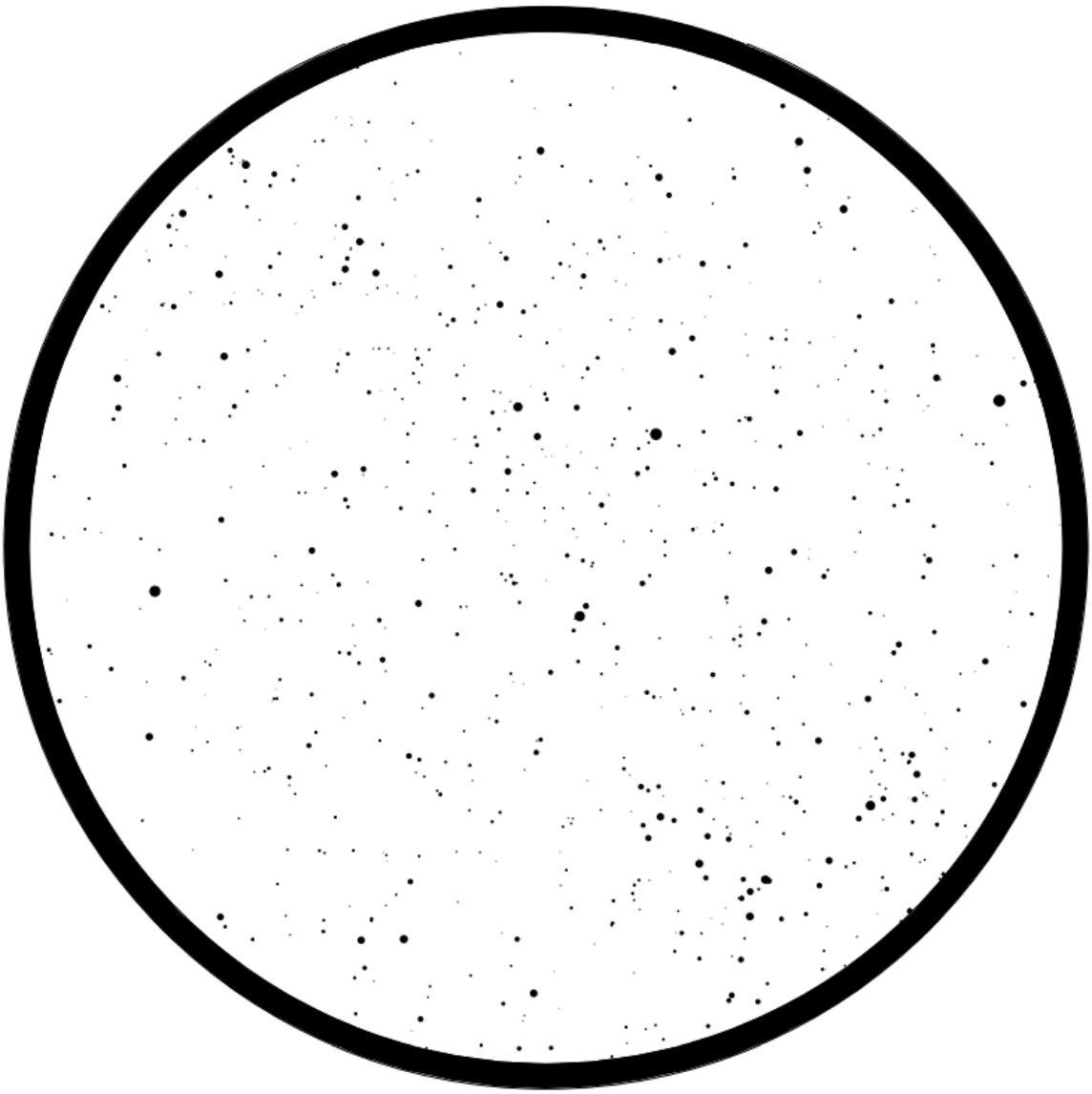
    (b) 1.56 m

    (c) 4.22×10^{7} m

    (d) 3.58×10^{7} m

    (e) None of the above

11. Four bright stars were erased from the sky map below. All of these stars have magnitudes lower than 3.0. Select the alternative with the four stars that were erased from the sky map.



(a) α And, α CrB, β Her, and α PsA.

(b) α And, α Aql, β Cet, and α Lyr.

(c) α Aql, α CrB, β Her, and α Peg.

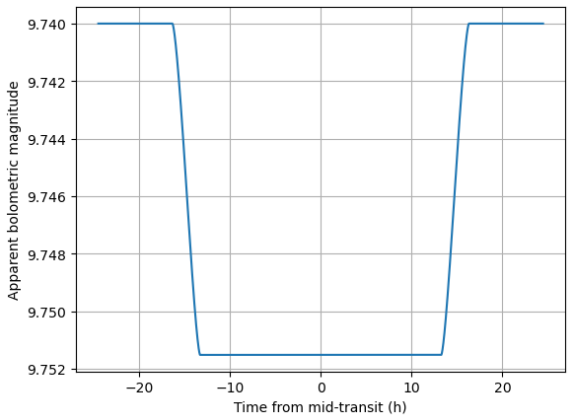
(d) β Cet, β Her, α Peg, and α Oph.

(e) α Aqr, α CrB, α Cyg, and α Her.

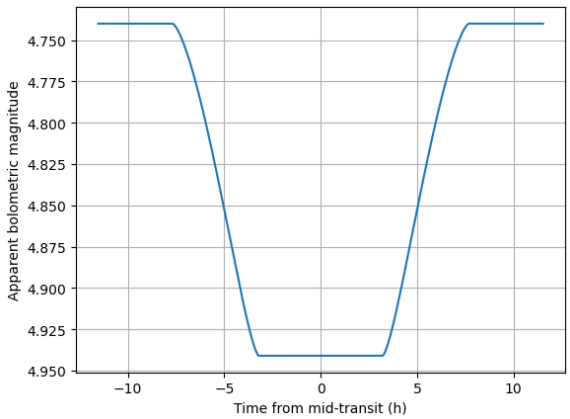
12. An extraterrestrial form of life from a distant planetary system observed a transit of Jupiter across the Sun. The transit was central, which means that the center of Jupiter's disk intersected with the center of the Sun's disk throughout the transit. Neglecting the effect of limb darkening, which of the light curves below could correspond to the transit of Jupiter seen by the extraterrestrial?

    The x-axis on the plots corresponds to the time difference in hours from the middle of the transit, and the y-axis corresponds to the apparent bolometric magnitude of the star.

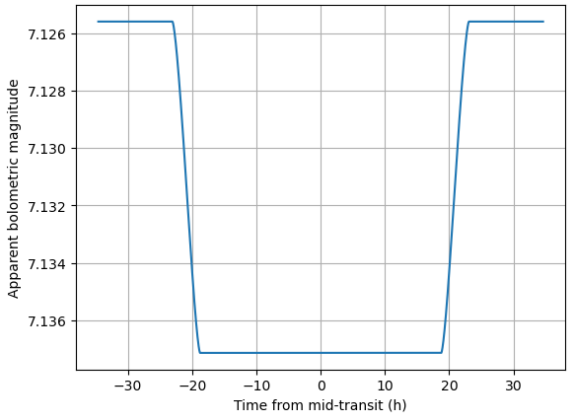
    (a)



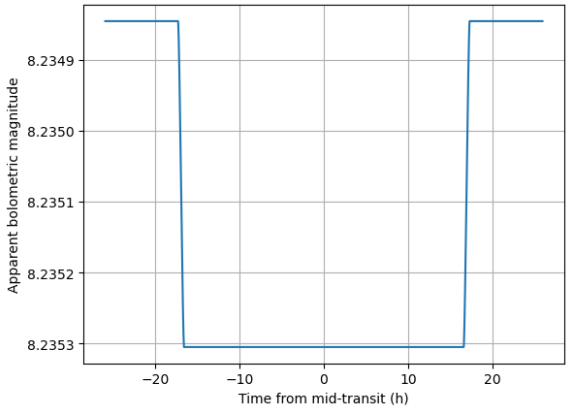
    (b)



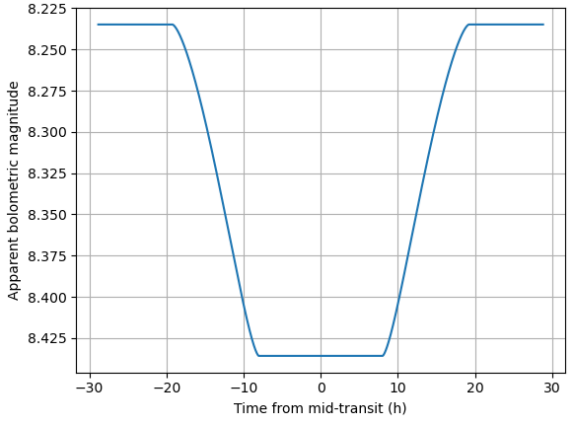
(c)


Apparent bolometric magnitude
7.126
7.128
7.130
7.132
7.134
7.136
−30
−20
−10
0
10
20
30
Time from mid-transit (h)

(d)


Apparent bolometric magnitude
8.2349
8.2350
8.2351
8.2352
8.2353
−20
−10
0
10
20
Time from mid-transit (h)

(e)


Apparent bolometric magnitude
8.225
8.250
8.275
8.300
8.325
8.350
8.375
8.400
8.425
−30
−20
−10
0
10
20
30
Time from mid-transit (h)

13. What is the smallest angular separation between Algol (β Per) and the Sun throughout the year? Algol's declination is $+40^\circ 57'$, and its right ascension is 3h08min.

    (a) $11^\circ 49'$.

    (b) $15^\circ 45'$.

    (c) $22^\circ 26'$.

    (d) $26^\circ 56'$.

    (e) $29^\circ 03'$.

14. Consider a cloud of hot gas with radius R and total (electromagnetic) power emission rate P. Keeping the average temperature of gas particles and average emissivity per particle constant, the cloud now expands to a radius $2R$. How does the net total power P radiated away, observed from far away, change in the cases where the cloud is optically thin (optical depth $\tau \ll 1$) and optically thick (optical depth $\tau \gg 1$), respectively? Assume the total number of particles remains constant.

    (a) Remains approximately constant, remains approximately constant

    (b) Remains approximately constant, increases significantly

    (c) Increases significantly, decreases significantly

    (d) Increases significantly, remains approximately constant

    (e) Increases significantly, increases significantly

15. Lunar laser ranging experiments show that, due to tidal effects, the semi-major axis of the Moon's orbit increases by around 38 mm per year. Assuming that the recession rate is constant, by approximately how much does this recession increase the Moon's sidereal orbital period each year? The mass of the Moon is 7.3×10^{22} kg. *Hint:* $(1+x)^\beta \approx 1 + \beta x$ *for* $|x| \ll 1$.

    (a) 230 μs

    (b) 350 μs

    (c) 380 μs

    (d) 690 μs

    (e) 230 ms

16. Kepler-22b is an exoplanet discovered by transiting with an orbital period of 290 days. Suppose it has a mass of 4.84×10^{25} kg and a moon similar to Earth's, with a prograde orbit near the ecliptic plane of the Kepler-22 system and a semi-major axis of 7.70×10^8 m. What is the length of a synodic lunar month, i.e., the time between successive full moons, for an observer on Kepler-22b? You can neglect the mass of the moon.

    (a) 25.0 days

    (b) 27.3 days

    (c) 29.5 days

    (d) 30.2 days

    (e) 33.6 days

17. A quickly-moving black hole passes through the center of a large star cluster. While the black hole does not experience any direct collisions with any stars in the cluster or significantly disrupts any stellar atmospheres, the black hole imparts a fraction of its momentum to several stars in the cluster close to the black hole's trajectory. (This is known as dynamic gravitational friction and can be explained purely with Newtonian mechanics.) Some time passes, and due to complex multi-body interactions, the additional energy and momentum get distributed over the entire star cluster.

    After a sufficiently long time, which of the following statements are likely TRUE regarding the system (compared to before the black hole event), assuming no further perturbations occur?

    (I) The average peak blackbody emission wavelength among stars in the cluster decreases significantly.

    (II) Stars are on average further apart from each other than they were before.

    (III) Stars have lower relative velocities with respect to each other than they had before.

    (a) I only
    (b) II only
    (c) III only
    (d) II and III
    (e) I and II

18. The most distant known galaxy, MoM-z14, was recently observed by the James Webb Space Telescope and has a redshift of $z = 14.44$. If the current cosmic microwave background (CMB) temperature is 2.725 K, what was the CMB temperature at MoM-z14?

    (a) 0.1887 K
    (b) 39.35 K
    (c) 42.07 K
    (d) 568.2 K
    (e) 3208 K

19. Assume a spherical star has a density profile described by the equation below:

$$\rho(r) = \left(1 - \frac{r^3}{R^3}\right)\ \rho_c, \quad 0 \leqslant r \leqslant R$$

    Find ρ_c in terms of the total mass, M, and the radius of the star, R.

    (a) $\frac{3M}{\pi R^3}$
    (b) $\frac{15M}{8\pi R^3}$
    (c) $\frac{3M}{2\pi R^3}$
    (d) $\frac{21M}{16\pi R^3}$
    (e) $\frac{6M}{5\pi R^3}$

20. Some stars formed in the early universe are still observable today. If an observer observed one and it is still in the main sequence phase, what would most likely be its mass?

    (a) $< 2\ M_\odot$

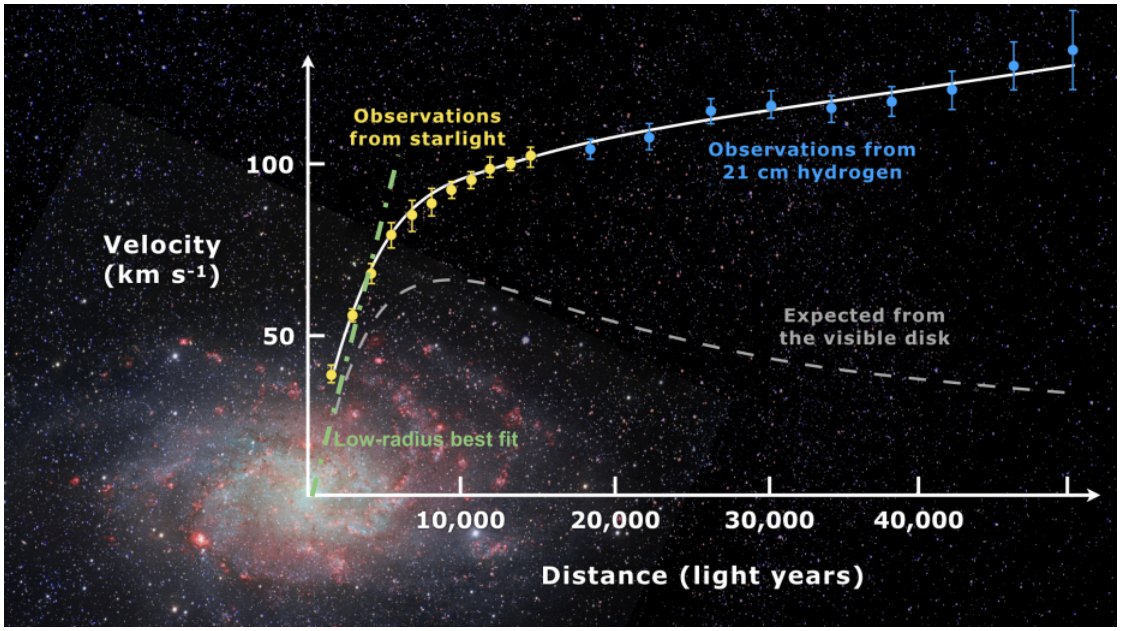
(b) $2 - 8\ M_\odot$

(c) $8 - 12\ M_\odot$

(d) $12 - 20\ M_\odot$

(e) $> 20\ M_\odot$

Use the following information for Questions 21, 22, and 23. Consider the following graph, which shows the mean orbital velocity of stars around a galaxy's center as a function of distance from the center. The graph also shows the expected orbital velocities (dashed line) from observations of visible matter. The discrepancy between the observed and expected values is assumed to be due to invisible dark matter.



21. Consider orbital radii within the most luminous part of the disk, at around $r \leqslant 5000$ ly. Let $M(r)$ be the total mass (of all forms) enclosed within a distance r from the galactic center. Approximating the rotation curve by the low-radius best fit (dot-dash) line, we can infer that $M(r)$ is roughly proportional to r^n for some value of n (over small r).

In light of the value of n, and for $r \leqslant 5000$ ly, which of the following hypotheses is **MOST** consistent with the data?

(a) Near the galactic center, the mass is concentrated in and is nearly uniform over the 2D galactic plane.

(b) The majority of the contribution to $M(r)$ comes from the existence of a supermassive black hole at the galactic center.

(c) Dark energy-based repulsion from the galactic center induces a mass distribution of the form $\rho(r) \propto r^2$.

(d) Complex gravitational interactions with the dark matter halo make such power laws impossible, with no single value of n being consistent with the best fit line.

(e) Gas, dust, and stars are approximately uniformly distributed in 3D space near the galactic center with no significant radius dependence of density.

22. Now, for larger values of r, the discrepancy between the observed and expected rotation curves begins to appear more significant. Assume that the expected rotation curve (dashed gray) curve is calculated off the distribution of all baryonic ("standard") matter, while the observed rotation curve (solid white) is due to the combination of baryonic matter and dark matter. Within a radius of $r = 40,000$ ly from the galactic center, and according to the graph, which of the following is closest to the ratio between the dark matter mass and the baryonic matter mass enclosed?

    (a) 2

    (b) 5

    (c) 10

    (d) 25

    (e) 50

23. Consider the following three assertions related to the techniques used to measure the overall rotation curve, in the context of the aforementioned figure:

    (I) Interstellar dust is colder at lower values of r, making the dust obscure less starlight.

    (II) O/B type stars in the spiral arms of galaxies emit significant quantities of UV light, ionizing nearby H-I clouds and thus inhibiting the emission of 21-cm radiation in their vicinities.

    (III) The lower star number density for higher values of r means that the stars that do exist typically move at relativistic speeds, complicating Doppler-based stellar velocity measurements.

    Which of the above are **TRUE**?

    (a) I only

    (b) II only

    (c) III only

    (d) I and II only

    (e) None of the above

24. Suppose the Honeyhive galaxy and the Gold Leaf galaxy are diametrically opposite when viewed from the Earth, and the following sequence of events happens:

    (a) A type Ia supernova happens in the Honeyhive galaxy.

    (b) Astronomers on Earth measure the redshift of (the spectral lines) in the Honeyhive supernova to be $z = 0.5$. At the same time, another supernova happens in the Milky Way galaxy.

    (c) Aliens in the Gold Leaf galaxy measure the redshifts of the the supernovae in the Honeyhive and Milky Way galaxies.

    If the alien astronomers measure the redshift of the Milky Way supernova to be $z = 0.8$, what redshift would they measure for the Honeyhive supernova?

    You may assume that the local velocities of the galaxies are negligible, that the universe is simple homogeneous, and isotropically expanding, and that redshift depends only on the scale factor and is the same function of distance for all observers.

    (a) 0.2

(b) 0.4

(c) 1.3

(d) 1.7

(e) 2.7

25. The Balmer-α, H-I, and Lyman-α lines are important spectral lines in astronomy, all from hydrogen.

    - The Balmer-α line occurs when an electron in a hydrogen atom goes from the $n = 3$ state to $n = 2$.
    - The H-I line occurs from an electron in the 1s shell of a hydrogen atom undergoing a spin-flip transition between two hyperfine levels.
    - The Lyman-α line is the emission line that occurs when an electron in a hydrogen atom goes from $n = 2$ to $n = 1$.

    Order these lines from the *shortest* to *longest* wavelength.

    (a) Balmer-α, H-I, Lyman-α

    (b) H-I, Balmer-α, Lyman-α

    (c) Lyman-α, H-I, Balmer-α

    (d) Lyman-α, Balmer-α, H-I

    (e) Balmer-α, Lyman-α, H-I

26. Star X and star Y form an eclipsing binary system, where star X is larger, and fully blocks star Y during an eclipse. During the primary eclipse, the apparent magnitude of the binary system increases by 0.200, and during the secondary eclipse, the apparent magnitude only increases by 0.100. If T_X is the surface temperature of star X and T_Y is the surface temperature of star Y, which of the following is a possible value of T_X/T_Y? Ignore limb darkening.

    (a) 0.679

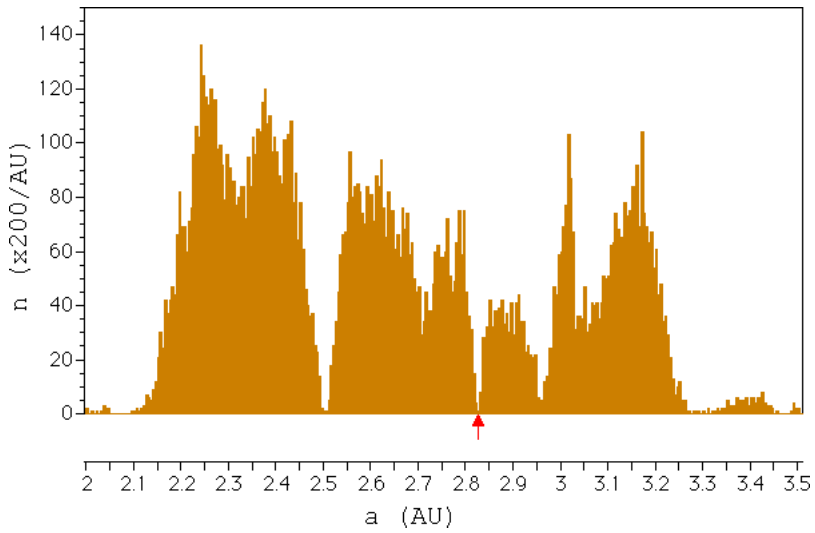
    (b) 1.000

    (c) 1.176

    (d) 1.414

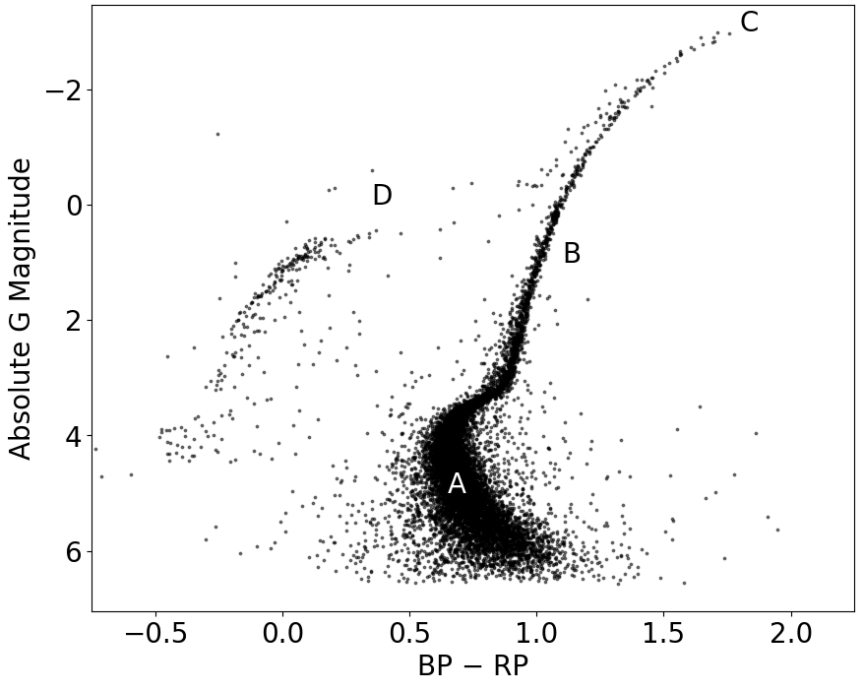
    (e) 2.000

27. When plotting the distribution of asteroids in the asteroid belt by their semi-major axis, we can see several regions, known as "Kirkwood gaps," where asteroids' orbits are unstable due to an orbital resonance with Jupiter ($a = 5.20$ AU). What orbital resonance is responsible for the gap marked with the arrow?



(a) 8:5
(b) 11:6
(c) 9:4
(d) 5:2
(e) 3:1

28. Below is an HR diagram for a globular cluster from Gaia DR3 data:



The figure has labeled regions: "A", "B", "C", and "D". 1) Identify the region where stars undergo hydrogen burning in their cores. 2) Identify the region where stars experience *Helium flash*.

(a) 1. Region A; 2. Region B

(b) 1. Region A; 2. Region C

(c) 1. Region B; 2. Region C

(d) 1. Region B; 2. Region D

(e) 1. Region C; 2. Region D

29. Which of the following is closest to the ratio of the number of photons from the Sun that strike the Earth each second to the number of photons from the Cosmic Microwave Background that strike the Earth each second? *(Assume the Cosmic Microwave Background to be a uniform and isotropic blackbody signal at $T_c \approx 2.725\,\mathrm{K}$. As a hint, you may approximate all photons from a given source as being at the peak emission energy.)*

(a) 5×10^4

(b) 2×10^5

(c) 3×10^7

(d) 1×10^8

(e) 4×10^8

30. Consider two galaxies:

- Galaxy S is a spiral galaxy with blue spiral arms.
- Galaxy E is an elliptical galaxy with no blue stars, only red stars.

Which of the following statements are likely TRUE?

(I) We can find planetary nebulae in both galaxies.

(II) Galaxy S is likely to have ongoing star formation.

(III) Galaxy E is likely to have ongoing star formation.

(IV) Type II supernovae are more likely to be found in Galaxy S

(V) Type Ia supernovae are likely to be found only in Galaxy E

(a) (I), (II), (III)

(b) (I), (II), (IV)

(c) (I), (III), (IV)

(d) (I), (IV), (V)

(e) (II), (IV), (V)